

PRINCIPLES OF FLUID FLOW
AND
SURFACE WAVES
IN
RIVERS, ESTUARIES, SEAS AND OCEANS
(Edition 2011)

Leo C. van Rijn



Universiteit Utrecht

Physical Geography



Other publications:

Principles of Sediment Transport in Rivers, Estuaries and Coastal Seas by Leo C. van Rijn, 1993 and 2006 (update)

*Principles of Coastal Morphology,
by Leo C. van Rijn, 1998*

*Principles of Sedimentation and Erosion Engineering in Rivers, Estuaries and Coastal Seas
by Leo C. van Rijn, 2005*

Manual Sediment Transport Measurements in Rivers, Estuaries and Coastal Seas by Leo C. van Rijn, 2007 (update)

**Aqua Publications
The Netherlands
(WWW.AQUAPUBLICATIONS.NL)**

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AQUA PUBLICATIONS

Published in The Netherlands

- first published 1990 (Bound Edition)

- last update 2011 (Bound Edition)

Aqua Publications

The Netherlands

(WWW.AQUAPUBLICATIONS.NL)

CIP-DATA KONINKLIJKE BIBLIOTHEEK, DEN HAAG, THE NETHERLANDS

Rijn, Leo C. van

Principles of fluid flow and surface waves in rivers, estuaries, seas and oceans

Author: Leo C. van Rijn

Publisher: Aqua Publications (www.aquapublications.nl)

ISBN: 978-90-79755-02-8

NUR: 929, mechanics

Subject headings: Fluid mechanics

Photographs Cover

Yangtze river, China

Egmond coast, The Netherlands

Ocean wave, Hawaii

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FOR THOSE WHO LIKE WATER

**PRINCIPLES OF FLUID FLOW AND SURFACE WAVES
IN RIVERS, ESTUARIES, SEAS AND OCEANS**

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PREFACE 1

This book reflects a one-year lecture course of fluid flow and surface waves for physical geographers of the University of Utrecht in The Netherlands. The book deals essentially with the basic principles of fluid mechanics in rivers, estuaries, seas and oceans.

The early Chapters 2, 3 and 4 cover the field of fluid properties, hydrostatics and kinematics. Chapter 5 on fluid dynamics describes the momentum equations of Euler, Bernoulli, Navier-Stokes and finally Reynolds, who introduced a time-averaging method to deal with turbulent flow. Chapter 6 deals with steady uniform flow in rivers. Phenomena like subcritical and supercritical flow, smooth and rough flow, boundary layer flow and flow resistance are explained. Work and power related to fluid dynamics are also explained.

Chapter 7 presents steady non-uniform flow in rivers giving information of potential flow, gradually varied and rapidly varied flow, curved flow and flow-induced forces (drag and lift).

The last two Chapters 8 and 9 deal with non-steady flow related to long and short surface waves.

Long wave phenomena like progressive waves, standing waves, translation waves, river flood waves, tidal waves, density-induced-waves and storm surge waves are described in detail.

Basic short wave properties are presented in Chapter 9. Phenomena like shoaling, refraction, diffraction and breaking are explained. Phenomena of the wave boundary layer and its effect on a current are also presented. The generation of longshore currents inside the surf zone is described. Finally, random waves are explained. The book ends with Appendices on basic formulae, mathematics, turbulence, methods to solve the flow equations and model scaling laws.

Basic knowledge of mathematics (especially differential equations) is required to understand the derivation of the equations of continuity and motion which appear throughout the text. To fresh up the readers memory, the most essential information of mathematics with respect to fluid mechanics is presented in Appendix B.

The book has been written with a view to sediment transport and morphology. Both fields of work are described extensively in: "Principles of Sediment Transport in Rivers, Estuaries and Coastal Seas" and "Principles of Coastal Morphology".

The author hopes that the present book will serve as a useful tool for scientists, engineers and students in civil engineering, earth sciences, physical geography and oceanography.

Professor Leo C. van Rijn, 1993, 2011

PREFACE 2

In his long career as a researcher, Professor Leo van Rijn has given a major boost to sediment transport research. The transport formulae he derived are used all over the world and many researchers and professionals are familiar with his books on coastal sediment transport and morphology. Moreover, he has led a number of landmark research programs, national as well as international, with sediment transport as a common denominator.

On top of this, he has tested his knowledge and trained a large number of young colleagues in a wide range of practical application projects, all with a sediment transport component. He has done this first under the umbrella of WL|Delft Hydraulics, and after the merger of WL into Deltares in 2008 under the umbrella of Deltares. Many clients have profited from his sometimes unconventional views and insights.

But this is not his only career. He also serves since many years as a professor at Utrecht University, training generations of students in Physical Geography with his courses and exercises in hydrodynamics and sediment transport. Without exaggerating, one may claim that he has brought these courses and exercises to a substantially higher level.

Now that he is about to retire, Leo has found the time for an extensive write-up of his course material on coastal hydrodynamics, enriched with many insights he has gathered over the years. The result is this impressive book, which in my opinion is a must for every researcher in coastal dynamics. It gives an almost complete overview of the aspects of hydrodynamics that are necessary to understand coastal dynamics.

Thorough knowledge of the water motion due to waves and currents on a coast, including their complex interactions, is indispensable to understand how a coast behaves. This knowledge ranges from classical textbook material through to state-of-the-art research results.

Maybe the greatest value of this book lies in the fact that it covers this entire range. This makes it a goldmine to people entering this fascinating field, be it students or young professionals, and an invaluable reference to the more experienced. I congratulate Leo with this laudable initiative. Deltares is proud to have had him among its staff and happy to have been able to support the production of this book.

Professor Huib de Vriend

Director Science Deltares, Delft, The Netherlands, 2011

Acknowledgements

The author wishes to acknowledge the Deltares Organization and the Department of Physical Geography of the University of Utrecht for providing the financial means to publish this 2011 Edition on the occasion of my retirement.

I am also grateful to the following copyright holders for permission to reproduce figures: Dr. M. de Vries and Mr. C. Verspuy of Delft Technical University, Mr. R. Thabet of Delft Hydraulics, American Society of Civil Engineers, MacMillan Press, Pitman Press and Longman Group.